Exercise 213

For the following exercises, evaluate the functions. Give the exact value.

$$\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$$

Solution

The inverse cosine gives an angle between 0 and π.

$$x = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$\cos x = \frac{\sqrt{2}}{2}$$

The value of x that satisfies this equation is $\pi/4$. Now take the sine of $\pi/4$.

$$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

Therefore,

$$\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right) = \frac{\sqrt{2}}{2}.$$